17. $x=\int_0^y \sqrt{\sec^4 t-1}\,dt \qquad \frac{-\pi}{4}\le y\le\frac{\pi}{4}$

$$\frac{dx}{dy}=\sqrt{\sec^4 y-1}$$

$$\left(\frac{dx}{dy}\right)^2=\sec^4 y-1$$

$$L=\int_{-\frac{\pi}{4}}^{\pi/4}\sqrt{1+\sec^4 y-1}\,dy$$

$$L=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\sec^2 y\,dy=\tan y\Big|_{-\frac{\pi}{4}}^{\pi/4}=\tan\frac{\pi}{4}-\tan-\frac{\pi}{4}$$

$$=1--1=2$$

Jan 5-7:42 AM

19. $(1,1) \qquad L=\int_1^4\sqrt{1+\frac{1}{4x}}\,dx$

$$\left(\frac{dy}{dx}\right)^2=\frac{1}{4x}$$

$$\frac{dy}{dx}=\pm\frac{1}{\sqrt{4x}}=\pm\frac{1}{2}x^{-\frac{1}{2}}$$

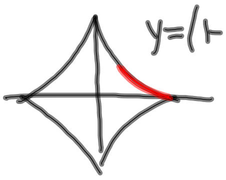
$y=\pm x^{\frac{1}{2}}+c \qquad y=\sqrt{x}$

Init. cond. $1=\pm 1+c \qquad y=-\sqrt{x}+2$

$c=0 \qquad c=2$

Jan 5-7:49 AM

22. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=1$ astroid



$y=(1-x^{2/3})^{3/2}$ $\frac{\sqrt{2}}{4}\le x\le 1$

$\frac{dy}{dx}=-\frac{2}{3}x^{-\frac{1}{3}}\cdot\frac{3}{2}(1-x^{2/3})^{\frac{1}{2}}$

$=\frac{-\sqrt{1-x^{2/3}}}{x^{1/3}}$

$\left(\frac{dy}{dx}\right)^2=\frac{1-x^{2/3}}{x^{2/3}}=\frac{1}{x^{2/3}}-1$

$\int_{\frac{\sqrt{2}}{4}}^{1}\sqrt{1+\frac{1}{x^{2/3}}-1}\,dx=\int_{\frac{\sqrt{2}}{4}}^{1}\frac{1}{x^{1/3}}\,dx$

$\int_{\frac{\sqrt{2}}{4}}^{1}x^{-\frac{1}{3}}dx=\frac{3}{2}x^{2/3}\Big|_{\frac{\sqrt{2}}{4}}^{1}$

$=\frac{3}{2}\left(1-\left(\frac{\sqrt{2}}{4}\right)^{2/3}\right)=\frac{3}{4}$

$L=8\cdot\frac{3}{4}=6$

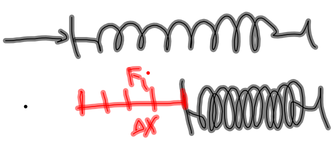
Jan 5-7:56 AM

7.5a Applications from Science and Statistics

Work done by a variable force: $\boxed{W=\int_a^b F(x)\,dx}$

work done by a constant Force:

$W=F\cdot d$



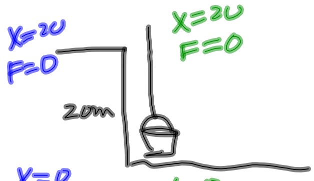
$\Delta W=F_i\,\Delta x$

$W\approx\sum F_i\,\Delta x$

$W=\lim_{\Delta x\to 0}\sum F_i\,\Delta x=\int_a^b F\,dx$

Dec 17-7:20 PM

A leaky bucket weighs 22N empty. It is lifted from the ground at a constant rate at a point 20m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70N of water but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done.



$x=0$, $F=70$

$x=0$, $F = .4\frac{N}{m}\cdot 20m = 8N$

slope: $\frac{0-70}{20-0} = -\frac{7}{2}$

$F = -\frac{7}{2}(x-0)+70$

$F = -\frac{7}{2}x+70$

rope slope: $\frac{0-8}{20-0} = -\frac{8}{20} = -\frac{2}{5}$

$F = -\frac{2}{5}x+8$

bucket: $W = 22N\cdot 20m$

$W = 440\,Nm$

$W = 440\,J$

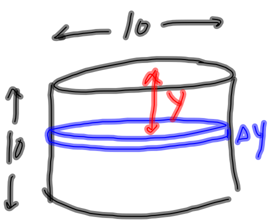
water: $\int_0^{20} -\frac{7}{2}x+70\,dx = 700\,J$

rope: $\int_0^{20} -\frac{2}{5}x+8\,dx = 80\,J$

Total: $440 + 700 + 80 = 1220\,J$

Dec 17-7:25 PM

How much work does it take to pump all the water over the rim of a cylindrical tank of height 10ft and diameter 10ft?



work for 1 slice

$\Delta W =$ weight · distance

distance $= y$

weight = density · volume
$\left(\frac{lb}{ft^3}\right)$ ft^3

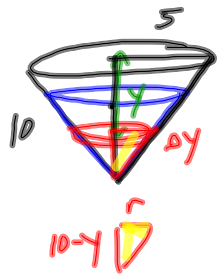
$\Delta W = 62.4\cdot\pi\cdot 5^2\cdot\Delta y\cdot y$ $= 62.4\frac{lb}{ft^3}\,\pi\cdot 5^2\cdot\Delta y$

$W = \lim_{\Delta y\to 0}\sum 62.4\pi\cdot 5^2 y\,\Delta y$

$W = \int_0^{10} 62.4\,\pi\cdot 5^2\cdot y\,dy = 245{,}044$ ft-lbs

Dec 17-7:29 PM

A conical tank of height and diameter 10ft is filled to within 2 ft of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank?



$\Delta W =$ weight $\cdot$ distance

$= 57 \cdot \pi r^2 \Delta y \cdot y$

$\Delta W = 57 \cdot \pi \left(\frac{10-y}{2}\right)^2 y \, \Delta y$

$W = \int_2^{10} 57 \cdot \pi \left(\frac{10-y}{2}\right)^2 \cdot y \, dy$

$\frac{5}{10} = \frac{r}{10-y}$

$r = \frac{10-y}{2}$

Dec 17-7:31 PM